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COAGULATION OF AEROSOL PARTICLES IN A TURBULENT ATMOSPHERE

Ye. N. Teverovskiy
Submitted by Academician A. N. Kolmogorov
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I. DIFFUSION OF SMOKE IN THE GROUND LAYER OF THE ATMOSPHERE

The intensive dispersion of any admixture introduced into the atmosphere is explained by the turbulence of the atmosphere.

The irregularity of turbulent motion means that the problem of the disper-sion of aerosol particles suspended in the atmosphere is solved by statistical methods in the same manner as the problem of vortical diffusion. In solving this problem, it is assumed that the introduction of aerosol particles into the atmosphere does not change the properties of the atmosphere.

Taylor [1], Richardson [2], and Schmidt [3] generalized the problem of molecular diffusion in the case of vortical diffusion.

The concentration of the substance introduced into the atmosphere is de-termined from the equation of diffusion

$$\frac{\partial c}{\partial c} + u\frac{\partial c}{\partial x} = \frac{\partial}{\partial x}\left(D_x \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_y \frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial z}\left(D_z \frac{\partial c}{\partial z}\right); \qquad (1)$$

Here c is the gravimetric concentration; x is the mean longitudinal wind direc-tion; y is the direction perpendicular to this; z is the vertical direction; u is the mean speed of the wind; and D_x, D_y, and D_z are the components of the coefficient of vortical diffusion D.

The first solution of this equation was given by Roberts [4], who con-sidered the coefficient of diffusion constant.

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For an instantaneous point source (a puff of smoke) occurring at a moment $t = 0$ at a point $x = y = z = 0$, Roberts obtained

$$C = \frac{P}{(8\pi Dt)^{1/2}} \cdot e^{-\frac{(x-ut)^2 + y^2 + z^2}{4Dt}} , \tag{2}$$

where t is the time from the moment of intorduction of the substance into the atmosphere, and P is the weight of the substance introduced.

For a continually active point source

$$C = \frac{g}{4\pi Dx} \cdot e^{-\frac{u(y^2+z^2)}{4Dx}} , \tag{3}$$

where g is the rate (discharges per second) of the source.

For a continually active linear source of infinite length

$$c = \frac{Q}{2(\pi Dux)^{1/2}} \cdot e^{-\frac{uz^2}{4Dx}} \tag{4}$$

where Q is the discharge per second per unit of length of the source.

Richardson [5] showed that the coefficient of turbulent diffusion varies within wide limits, and that its magnitude depends on the scale of the phenomenon in question. This is explained by the fact that eddies of varying magnitude exist in a turbulent atmosphere. As a result, the magnitude of the coefficient of diffusion increases with the scale of the phenomena observed. Therefore Roberts' assumption of the constancy of this coefficient in a turbulent atmosphere is incorrect.

Richardson showed that if one examines the dispersion of particles over a certain interval of time at an average distance λ, then the coefficient of diffusion for this scale of dispersion is

$$D = a \cdot (\lambda)^{4/3} \tag{5}$$

The magnitude of a as determined by Richardson is equal to 0.2 cm 2/3/sec.

Sitton [6], taking Richardson's concepts as a basis, considered that the coefficient of turbulent diffusion is related to the time of movement of the cloud.

When the volume of the released cloud is small, it will be dispersed by small eddies which are roughly commensurate with the dimension of the cloud.

In proportion to the increase of the cloud, all larger eddies will participate in its dispersion.

On the basis of Taylor's correlation theory, Sitton shows a relation between the coefficient of diffusion and the time of movement of the cloud:

$$D = \frac{S^2}{4} u^m t^{m-1} ; \tag{6}$$

Here S is the constant specific intensity of diffusion; m is a dimensionless parameter related to the vertical distribution of the temperatures and varying from 1 to 2 (with the isotherm $m \approx 2$).

For concentration from an instaneous point source, Sitton obtains the following expression:

$$C = \frac{P}{\pi^{3/2} S^3 (ut)^{3/2 m}} \cdot e^{-\frac{(x-ut)^2 + y^2 + z^2}{S^2 (ut)^m}} . \tag{7}$$

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For a continually active point source

$$C = \frac{P}{\pi S^2 u_x m} \, e_{,o} \, \frac{y^2 + z^2}{S^2_x m}.$$ (8)

For a continually active linear source of infinite length

$$C = \frac{Q}{\pi^{1/2} S_u x^{1/2} m} \cdot e^{-\frac{z^2}{S^2_x m}}.$$ (9)

For a continually active linear source with width $2y_0$

$$C = \frac{Q}{2\pi^{1/2} S_u x^{1/2} m} \left[\Phi\left(\frac{y_0 - y}{S_x m^{1/2}}\right) + \Phi\left(\frac{y_0 + y}{S_x m^{1/2}}\right) \right] \cdot e^{-\frac{z^2}{S^2_x m}}$$ (9a)

where $\Phi(y)$ is Cramp's function.

Laykhtman [7], starting with the hypothesis that the magnitude of the coefficient of turbulent diffusion in the ground layer of the atmosphere will depend on the height, determined this relation from the equation for the "mixing length" which he proposed in a new form:

$$l_z = \varkappa \left(\frac{z}{z_o}\right)^{1 - 1/p} \cdot z_0)$$ (10)

where l_z is the "mixing length"; x is a constant equal to 0.38; z is the height above the surface of the earth; z_0 is the roughness of the surface (the height above the surface of the earth at which the speed of the wind is assumed equal to zero); p is the parameter characterizing the stability of the atmosphere vertically, in which p varies from 1 to ∞.

With an adiabatic vertical distribution of temperatures (isotherms) r = ∞; equation (10) develops into the usual formula of Prandtl for the "mixing length" in the flow of an incompressible fluid, close to the boundary.

$$l = \varkappa z$$ (10a)

The magnitude of parameter p may be determined from Laykhtman's formula for the distribution of speeds of wind by height:

$$\frac{u_z}{u_1} = \frac{z^{1/p} - z_o^{1/p}}{z_1^{1/p} - z_o^{1/p}}.$$ (11)

The relation of the vertical component of the coefficient of turbulent diffusion D_z to the height is presented by Laykhtman in the form

$$D_z = \frac{\varkappa^2 u_1 z^{\frac{p-1}{p}}}{p\left[\left(\frac{z_1}{z_o}\right)^{1/p} - 1\right]} \cdot z_c^{1/p} = D_{z_1}\left(\frac{z}{z_1}\right)^{\frac{p-1}{p}},$$ (12)

where D_{z_1} is the coefficient of turbulent diffusion at height z_1. Equation (12) is true only for the ground layer of the atmosphere (approximately to a height of 10-15 meters).

The horizontal components of the coefficient of turbulent diffusion D_x and D_y are usually not considered to be related to the height and not equal in magnitude.

The equations obtained by Sitton were verified by us experimentally.

Smoke from a continually active linear source was introduced into the ground layer of the atmosphere, and its concentration at various distances was determined.

The results of the measurements showed that Sitton's formula (9) is essentially correct in its representation of the law for variation of concentration

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of smoke due to the intensity of the source, the speed of the wind, and the distance.

Together with Ya. L. Zabeshinskiy [8] we showed that in the case of a change of the average concentration at the center of the smoke wave at the ground—assuming that the surface of the earth will reflect the cloud of smoke, the concentration should be doubled—along the isotherm (m is assumed equal to two) equation (9) may be written in the form

$$c = \frac{2 Q}{\pi \, u_2 \, S_2 \, u_2 \, X} ;$$ (13)

The symbol S_z shows that only the dispersion upward is taken into consideration for a linear source; the symbol u_2 indicates that the speed of the wind was measured at a height of 2 meters.

Table 1 gives the values S_z (of the vertical component S), calculated from experimental data.

Table 1.   Values of Magnitude :

No of Experi- ment	Speed of Wind at Altitude of 2 m, in m/sec	Difference of Temperatures at Altitudes of 20 and 150 cm, in °C	Discharge of Source on a Meter Front per Minute, in G	Average Roughness z_0, in m	S_2
Level Terrain Without Grass					
1	5.2	—0.3	56	0.005	0.030
2	6.4	—0.9	16	0.005	0.026
3	7.4	+0.4	47	0.005	0.024
4	8.0	—0.2	18	0.005	0.023
Slightly Broken Terrain Covered With High Grass					
5	1.15	—0.4	18	0.1	0.086
6	2.8	0.0	18	0.1	0.094
7	3.2	—0.4	27	0.1	0.088
8	3.75	+0.1	27	0.1	0.092
9	3.9	+0.2	18	0.1	0.080
10	5.0	+0.4	18	0.1	0.090
11	5.9	—0.1	18	0.1	0.072

The roughness z_0 was determined from data on the distribution of the speed of wind according to height in accordance with equation (11); it was also assumed that for the isotherm $p = \infty$. In this case (11) takes the form

$$\frac{u_2}{u_1} = \frac{\ln \frac{z_2}{z_0}}{\ln \frac{z_1}{z_0}} .$$ (11a)

From this the value z_0 is also calculated.

From Table 1 it is evident that the magnitude S_z is essentially related to the degree of roughness z_0.

For the isotherm it may be assumed

$$S_z = \frac{D_z}{u_1 z_1} .$$

The value of D_z may be determined from equation (12), which takes the form for the isotherm

$$D_z = \frac{0.1444 \, u_1 z}{\ln \frac{z_1}{z_0}} .$$ (12a)

We have assumed $z = z_1 = 1$ m.

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The values of D_{z1}, calculated according to (12a) -- taking into consideration the fact that S_z was determined according to the data of the experiments in which the speed of wind was measured at a height of 2 m, must be increased to

$$\frac{u_2}{u_1} = \frac{\ln \frac{2}{z_0}}{\ln \frac{2}{z_0}}$$

when $z_0 = 10$ cm $\quad D_{z1} = 0.063\, u_1\, z_1$,
when $z_0 = 0.5$ cm $\quad D_{z1} = 0.027\, u_1\, z_1$.

Consequently, D_{z1} corresponds to the values S_z, given in Table 1.

The value of the horizontal component of the coefficient of diffusion $D_x = D_y$ may be found from the hypothesis that at a certain height above the earth the components of the coefficient of diffusion become the same: $D_x = D_y = D_z$, that is, an isotropic turbulence appears. (The determination of the magnitude of the horizontal component, which has been cited, was done together with D. L. Laykhtman and L. S. Borishanskiy.)

At this height the brake action of the surface of the earth ceases to affect D_z.

The measurements of vortical energy cited in Brent $[9]$ have been used for the determination of the height at which isotropic turbulence is reached.

In Figure 1 a curve is given showing the relation $\frac{v^2}{w^2}$ according to the data of various authors (ω is the component of vortical speed in a vertical direction, v, in a horizontal).
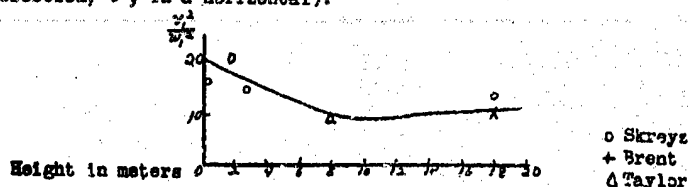

Height in meters

o Skreyz
+ Brent
Δ Taylor

Figure 1

As evident from the graph, the most probable assumption is that the height at which isotropic turbulence is reached along the isotherm will be equal to 12-14 meters. We assumed this height to be equal to 13 meters. At this height

$$D_x = D_y = D_{z\,(3)} = 13\, D_{z,\prime}$$

For roughness $z_0 = 5$ cm, $D_{z1} = 0.05 u_1 z_1$. Consequently, for this height

$$D_x = D_y = 0.45\, u_1 z_1.$$

It will be interesting to estimate the average value of the over-all coefficient of diffusion at a height of 1 meter (D_1). The magnitude D_1 may be calculated as the geometric mean $[4]$:

$$D = \sqrt[3]{D_x\, D_y\, D_z}. \tag{14}$$

In this case

$$D_1 = 5.54\, D_{z1} = \frac{0.8\, u_1 z_1}{\ln \frac{z_1}{z_0}}; \tag{14a}$$

When $z_0 = 10$ cm $\qquad\qquad D_1 = 0.35 u_1 z_1$
When $z_0 = 5$ cm $\qquad\qquad D_1 = 0.27 u_1 z_1$
When $z_0 = 0.5$ cm $\qquad\qquad D_1 = 0.15 u_1 z_1$

II. COAGULATION OF SMOKE PARTICLES IN A TURBULENT ATMOSPHERE

With the aid of ultramicroscopes specially adapted for field measurements, the change in magnitude of smoke particles in a cloud of smoke expanding in the ground layer of the atmosphere was investigated. The number of particles and the

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weight concentration of the smoke were measured at various points of the smoke wave. From the given measurements the average radius of the particles of smoke was calculated.

The measurements showed that the average radius of the particles increases quickly in proportion to the advance of the smoke cloud, and increases from 2 to $4 \cdot 10^{-5}$ cm upon reaching a distance of 1,000 meters.

The number of particles in the experiments was of the order of $10^4 \div 10^6$ per cc.

Two processes occur in the cloud of smoke moving in turbulent atmosphere: the dispersion of the particles and their coagulation.

According to Smolukhovskiy [10], the change of the number as a result of coagulation is equal to

$$-\frac{dN}{dt} = KN^2;$$

(15)

where N is the number of particles in a unit volume; t is the time; K is the constant of speed of coagulation.

On the other hand, the number of particles will decrease as a result of the turbulent dispersion of the smoke. According to (13)

$$N = \frac{A}{x};$$

(16)

where x is the distance traversed by the cloud on the wind; $A = \frac{Q}{\frac{4}{3}\pi \frac{1}{2} S_2 \rho r^3 u}$ thus $N = \frac{c}{\frac{4}{3}\pi r^3 \rho}$; ρ is the density of the particles; r is their radius.

The decrease in number of particles with the distance will be equivalent to

$$\frac{dN}{dx} = -\frac{A}{x^2} = -\frac{1}{A}N^2.$$

(17)

Assuming that x = ut we modify equation (15):

$$-\frac{dN}{dx} = \frac{K}{u}N^2.$$

(18)

Assuming that the processes of dispersion and coagulation proceed independently of one another, the total decrease in the number of particles may be expressed by the equation

$$-\frac{dN}{dx} = \left(\frac{K}{u} + \frac{1}{A}\right)N^2.$$

(19)

Integrating (19) in the limits from x_0 to x and from N_0 to N, we obtain

$$\frac{1}{N} - \frac{1}{N_0} = \left(\frac{K}{u} + \frac{1}{A}\right)(x - x_0).$$

(20)

Hence for the constant of speed of coagulation we obtain:

$$K = u\left\{\frac{1}{x - x_0}\left(\frac{1}{N} - \frac{1}{N_0}\right) - \frac{\pi^{1/2} S_2 u c_0}{2 Q N_0}\right\}.$$

(21)

Equation (21) may also be expressed by the dimensions of the particles, namely

$$K = \frac{4}{3}\pi \rho u\left\{\frac{1}{x - x_0}\left(\frac{r^3}{c} - \frac{r_0^3}{c_0}\right) - \frac{\pi^{1/2} S_2 r_0^3 u}{2 \rho}\right\};$$

(22)

where r is the radius of the particles; c is the gravimetric concentration at distance x; r_0 and c_0, correspondingly, at distance x_0.

Table 2 shows the results of the calculations of the constant of speed of coagulation according to the data of our measurements.

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Table 2.   Values of K

No of p/p	Speed of Wind U_2, in m/sec	S_z	$K \cdot 10^8$ in cc /sec
1	1.15	0.086	0.5
2	2.8	0.094	19.1
3	3.2	0.088	10.0
4	3.75	0.092	13.4
5	5.0	0.09	188.0
6	5.9	0.072	177.0
7	5.2	0.03	4.5
8	6.4	0.026	10.8
9	7.4	0.024	16.4

As is apparent from Table 2, the constant of speed coagulation depends greatly on the speed of the wind and the vertical component of the coefficient of transfer (S_z).

This relation may be expressed by the following equation:

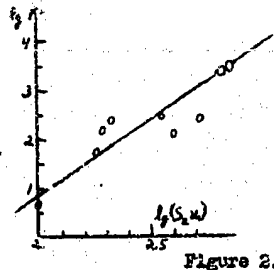In Figure 2 the relation of $\log K$ to $\log (S_z u)$ is represented

Figure 2.

The relation of K to S_z and U indicates that in turbulent atmosphere, in addition to coagulation produced by a thermal movement of molecules, there also occurs coagulation of particles produced by turbulent agitation.

The magnitude of the constant of speed of coagulation produced by molecular motion, according to the experiment of Whitely-Grey and Paterson [11], for particles with a radius of $2 - 3 \cdot 10^{-5}$ cm is approximately equal to $5 \cdot 10^{-10}$ cc/sec.

The minimum value of K found by us exceeds this by almost ten times; the maximum, by 4,000 times.

III. CALCULATION OF THE CONSTANT OF SPEED OF TURBULENT COAGULATION

The phenomenon of coagulation of particles in a flow was first investigated by Smolukhovskiy [10]. He examined coagulation in a laminar flow along a plane.

In this case as a result of varying speed of the flow at distance a from that plane, the number of collisions of particles increases, and the speed of coagulation

$$- \frac{dN}{dt} = \frac{3\lambda}{3} \cdot r^3 N^2 \frac{du}{dz} .$$ (23)

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Myuller [10] considered that in a flow in rotary motion the particles, from inertia, will diverge from the lines of flow, and the speed of coagulation will increase in proportion to the dimensions of the coagulating particles.

Wigand [12], assuming that the speed of turbulent agitation is proportional to the speed of the wind, and considering the factor of proportionality equal to 0.25, accepted the equations of Smolukhovskiy for the calculation of the speed of coagulation in a turbulent flow.

In this case (23) takes the form

$$- \frac{dN}{d\tau} = \frac{8}{3} r^3 N^2 u,$$

(24)

and the constant of speed of coagulation is also found to be proportional to the cube of the radius of the particles.

The calculation according the Wigand's formula for our case (the radius of particles of the order 3.10^{-5} cm) shows that the constant of speed of coagulation with a wind speed of 6 m/sec is equal to 5.10^{-11} cm/sec, that is, one-tenth of the constant of speed of molecular coagulation.

Like Sitton, we assumed that in turbulent agitation atmosphere eddies of random dimensions are formed, that is, that along with large eddies determined by diffusion, "microeddies" also exist. These small eddies (more truly, the mixing of small volumes of air at all points of space, produced by vortical movement) increase the number of collisions of particles, and determine the phenomenon of turbulent coagulation. Because of the irregularity of turbulent movement, an exact solution of the problem of turbulent coagulation may be obtained only by statistical methods; however, an attempt is being made in the present work to solve the given problem proceeding from the dimensions.

The constant of speed of coagulation of particles in a turbulent atmosphere must be equivalent to $K = K_1 - K_2$ (25) where K_1 is the constant of coagulation of particles produced by turbulent molecular movement and K_2 is the constant of coagulation of particles by turbulent movement.

Since the movement of particles in a turbulent flow is irregular, we may assume that in turbulent coagulation it will be determined by the coefficient of turbulent diffusion of the particles as in the problem of molecular coagulation. Consequently, using Smolukhovskiy's equation [10], which relates the constant of coagulation to the coefficient of diffusion, we may write

$$K = 4\pi r (D_1 + D_2),$$

(26)

where D_1 is the coefficient of diffusion of particles, depending on their respective transference produced by molecular movement, and D_2 is the coefficient of diffusion of particles depending on their respective transference produced by turbulent movement.

As indicated above, the coagulation of particles in a turbulent atmosphere due to mixing produced by turbulence is much greater than their coagulation due to the thermal movement of molecules.

Therefore, for a turbulent atmosphere it may be assumed that

$$K = 4\pi r D_2.$$

(26a)

It is also possible, using equation (26a), to estimate the average value of the coefficient of diffusion particles.

According to Table 2, with a wind of average force, of the order of 3 m/sec, $K = 20.10^{-8}$ cm^2/sec. Then, with the average radius of the particles $r = 3.10^{-5}$ cm, we find $D_2 = 0.6 \cdot 10^{-3}$ cm^2/sec.

The coefficient of diffusion of particles of given dimension, depending on molecular movement, is $D_1 \sim 10^{-6}$ cm^2/sec; that is, D_1 is smaller than D_2 by 10^3 times.

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The coefficient of turbulent diffusion of a flow in the ground layer of the atmosphere D, as was indicated above, is of the order of 10^4 cm^2/sec, and is therefore larger than D_2 by 10^7 times. The relation between the coefficient of diffusion of particles of smoke at a distance of the order of the distances between particles D_2, and the coefficient of diffusion of the total cloud of smoke D, may be found from the following:

It is evident that the larger D is, the larger D_2 must be; on the other hand, the coefficient of diffusion of particles must diminish with an increase in the kinematic viscosity of the medium, since an increase in the kinematic viscosity increases the dimensions of the "microeddies," and consequently reduces the relative transference of particles. Therefore, starting with the conception of dimension, we may write:

$$\tag{27}$$

where A is the dimensionless constant.

The expression for the constant of speed of turbulent coagulation will assume the following form:

$$K = 4\pi r A^n \cdot \frac{D^n}{v^{n-1}}. \tag{28}$$

Proceeding from the dimensions, we may also find the relation of the constant of speed of coagulation to kinematic viscosity.

The particles in a turbulent flow will collide only in the event that they are subjected to movements relative to one another from the direction of the flow. The smaller these movements are, the greater the probability that they will converge; for with large movements (larger than the distance between particles) all the particles will merely move together with the moving part of the flow without approaching one another. On the other hand, the greater the speed of movement produced by turbulent movements the greater the probability of a collision of particles.

Let us now examine the relation of the distance at which particles may converge and of their speed of movement to viscosity.

The distance S will be greater, the greater the viscosity; for the greater the viscosity, the greater will be the distance from one another and these movements will be extinguished, according to Reynolds criterion,

$$S \sim v. \tag{29}$$

The speed of movement of the particles will decrease with the viscosity according to Stokes' law:

$$v \sim \frac{1}{v}. \tag{30}$$

Thus, the time of collision of particles, that is, the time of their coagulation, is

$$t = \frac{S}{v} \sim v^2. \tag{31}$$

If one assumes that the constant of speed of coagulation is inversely proportional to the time

$$K \sim \frac{1}{t} \tag{32}$$

then the relation of K to V will be

$$K \sim v^{-2} \tag{33}$$

and, consequently, n = 3.

Hence (28) will assume the form $\quad K = 4\pi r A^3 \cdot \frac{D^3}{v^2}$ (34) that is, the constant of speed of turbulent coagulation is proportional to the cube of the coefficient of diffusion.

The value of the coefficient of turbulent diffusion may be calculated from (14a), and equation (34) finally assumes the form

$$K = 4\pi r A^3 \frac{\left(\frac{0.8 u_1 z_1}{\ln \frac{z_1}{z_n}}\right)}{v^2}. \tag{34a}$$

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The values of A are given in Table 3, where r is considered equivalent to 3.10^{-5} cm, and $a_1 = 1$ meter.

Table 3.   Values of A

No of p/p	Speed of Wind U_1, in m/sec	Roughness Z_0, in cm	Average Value of D^1, in m^2/sec	$K.10^8$ cc /sec	$A.10^5$
1	0.89	10	0.35	0.5	0.22
2	2.15	10	0.35	19.1	0.30
3	2.5	10	0.35	10.0	0.21
4	2.9	10	0.35	13.4	0.20
5	3.9	10	0.35	188.0	0.36
6	4.6	10	0.35	177.0	0.30
7	4.7	0.5	0.15	4.3	0.20
8	5.7	0.5	0.15	10.8	0.22
9	6.6	0.5	0.15	16.4	0.22

The average value of $A = 0.25 \cdot 10^{-5}$. The constancy of A may serve as a proof of the accuracy of the equations cited.

Equation (34) may also be obtained from an examination of the mechanism of a turbulent flow.

Contemporary ideas on the local structure of a turbulent flow were first formulated by Richardson $/5/$. It is assumed that the smallest dimension of pulsations in a turbulent stream λ, is determined by the viscosity of the medium on the condition that Reynolds' number Re λ, is of the order of unity $/13/$.

Dissipation of the energy of the turbulent flow occurs in these pulsations.

It is also assumed that turbulent pulsations cannot exist on a scale less than λ_o, since they must quickly be extinguished because of the viscosity; and at these scales only laminar (viscous) flow is possible.

Small scale pulsations, however, not only are extinguished but constantly regenerate, and, consequently they always exist in a turbulent flow.

The assumption of the presence of such pulsations may serve as an explanation of the causes of turbulent coagulation in a flow. Under the action of these fluctuations, the particles will be subjected to small movements at short distances $\lambda (\lambda \leqslant \lambda_o$. Inasmuch as these fluctuations are produced by the turbulence, one may assume that the coefficient of diffusion of particles even in this zone of magnitudes λ will obey the function found by Richardson $D_\lambda = 0.1(\lambda)^{4/3}$.

The calculation of D for λ of the order of the average distance between particles (with the average number of particles in our experiments $N = 5.10^5$ per cc, the distances between particles $\lambda = N^{-1/3} = 1.3.10^{-2}$cm) gives a value $D_\lambda = 0.5.10^{-3}$cm ^{2}/sec. This agrees well with the value of the coefficient of diffusion of particles D_2, calculated from the data for the coagulation of particles in the atmosphere.

The theory of local isotropic turbulence was worked out by Kolmogorov $/14/$ and Obukhov $/15/$. They showed that the dissipation of energy of a turbulent stream

$$\mathcal{E} \sim \frac{\rho(\Delta u)^3}{l},$$

(35)

where $\mathcal{E}$ is the dissipation of energy per unit of volume per unit of time; Δu is the change in speed of the flow at distance of the order 1; and 1 is the scale of the turbulence of the flow.

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Obukhov [13, 16] showed that the average quadratic movement of particles in a turbulent flow $\lambda (l \gg \lambda \gg \lambda_0)$

$$\overline{\lambda^2} \sim \frac{\varepsilon}{\rho} \cdot t^3 \qquad (36)$$

Let us use equation (35) for the dissipation of energy, assuming that D of the flow

$$D_\pi \sim \Delta u \cdot l, \qquad (37)$$

and let us substitute the value value Δu from (37) for (35). Then

$$\frac{\varepsilon}{\rho} \sim \frac{D_\pi}{l^4} \qquad (38)$$

or

$$\overline{\lambda^2} \sim \frac{D_\pi^3}{l^4} \cdot t^3. \qquad (39)$$

On the other hand, the average quadratic movement of a particle in irregular movement

$$\overline{\lambda^2} \sim D_y \cdot t. \qquad (40)$$

Inasmuch as the average quadratic movement of a particle in both cases will be the same, then it is evident from (39) and (40) that

$$D_y \sim \frac{D_\pi^3}{l^4} \cdot t^2. \qquad (41)$$

Inasmuch as D_{ch} is determined only by the viscosity of the medium, we may assume

$$\frac{l^4}{t^2} = b v^2, \qquad (42)$$

where b is the dimensionless constant.

Hence, we obtain the equation

$$D_y = b \cdot \frac{D_\pi^3}{v^2}, \qquad (43)$$

identical with equation (34) obtained earlier, where $A^3 = b$.

However, according to the theory of Obukhov, equation (43) may be true only in the case $\lambda \gg \lambda_0$.

The cubical relation of the constant of speed of coagulation to the coefficient of diffusion, which we found experimentally permits in our opinion the extension of the equations obtained in the theory of locally isotropic turbulence of Kolmogorov-Obukhov to scales less than λ_0, assuming that in these scales there actually exist pulsations of random dimensions produced by a turbulent flow.

One may also estimate the part of the energy present in the pulsations producing the coagulation of particles, using the values of the coefficients of diffusion of the flow and particles.

In conclusion let us examine constant A. It must characterize the scale of the phenomena observed.

Actually, the speed of coagulation is determined by the number of collisions of particles located at minute distances from one another. For these distances the coefficient of turbulent diffusion will be considerably less than the coefficient of turbulent diffusion D_p, calculated for the diffusion of the smoke cloud as a whole.

Denoting by D_y the coefficient of turbulent diffusion for scales of the order of distances between particles, we may assume:

$$A = \frac{D_y}{D_\pi}. \qquad (44)$$

For determination of D_y and D_π in the case of turbulent coagulation we may also assume:

$$D_y \sim \lambda \cdot \Delta u_\lambda \quad \text{and} \quad D_\pi \sim l \cdot \Delta u_l. \qquad (45)$$

From equation (35) it is obvious that

$$\Delta u_z \sim \left(\frac{\varepsilon}{\rho} z \right)^{1/3}. \qquad (46)$$

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Substituting equations (45) and (46) in (44), we obtain

$$A \sim \left(\frac{\lambda}{l}\right)^{2/3}$$

(47)

Magnitude λ is determined from the average distance between particles.

In our experiments the average distance between two particles was equivalent to $\lambda = 1.3 \cdot 10^{-2}$ cm.

For determining the magnitude l we use the equation of Laykhtman (10) for the "mixing length."

Because we are examining the case of adiabatic distribution of temperatures, the vertical component of the "mixing length" $l_z = \chi z$.

(10a)

At a height of 1 meter $l_1 = \chi \cdot 1 = 0.38$ meter.

It was indicated above that the average height at which isotropic turbulence is reached is equivalent to 13 meters. Therefore, we may assume

$$l = l_y = l_{13} = \chi \cdot 13 = 4.95 \, m.$$

(48)

Magnitude l_x for height of 1 meter may be determined in the same way as the average value of the coefficient of diffusion.

According to (4), $l = \sqrt[3]{l_x \, l_y \, l_z} = 2 \, m.$

Substituting the found values λ and l for (35), we find that $A = 0.3 \cdot 10^{-5}.$

(49)

Comparing the value A (A = 0.25·10⁻⁵) obtained from the experiment with the calculated value, we find that these magnitudes are of the same order.

Thus the expression for the constant of speed of turbulent coagulation assumes the following final form:

$$K = 4\pi r \left(\frac{\lambda}{l}\right)^{7} \frac{D^2}{r^2}.$$

(50)

This equation found relating the constant of speed of coagulation of particles in a turbulent medium to the coefficient of turbulent diffusion may be used to explain certain coagulation processes which take place in clouds and in the flocculation of colloidal particles in moving fluids.

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